



# Algorithmic Social Sciences Research Unit

ASSRU

Department of Economics  
University of Trento  
Via Inama 5  
381 22 Trento, Italy

DISCUSSION PAPER SERIES

1 – 2013/1

A JOURNEY THROUGH THE CORRIDORS OF A LABYRINTH\*  
BORGES' *LIBRARY*, SIMON'S *MAZES* & TURING'S *LIBRARIAN*

K. Vela Velupillai<sup>♥</sup>

JANUARY 2013

---

\*For most of my conscious life three activities have preoccupied my mind: logical puzzles (eg., the *Tower of Hanoi*), the construction of *meccano set* models and solving for maze paths in *labyrinths*. These three activities are predominant in the way I spend time with my children, whose curiosities, inspired by them, have – I hope – prevented my mind from death by atrophy. The ‘narrative’ in this paper is based on the fascination, and constructive inspiration, generated by puzzles, mazes in labyrinths and solving problems generated by them, on the minds and work of three of twentieth-century’s great thinkers: *Jorge Luis Borges*, *Herbert Simon* and *Alan Turing*, who, each in his own way, have made my intellectual life an unceasing adventure.

<sup>♥</sup>*Algorithmic Social Sciences Research Unit*, Department of Economics, University of Trento & Department of Economics, The New School of Social Research, New York. I am deeply grateful to Professor William Goldbloom Bloch, the author of the lyrical, poetic, **The Unimaginable Mathematics of Borges’ Library of Babel** (Bloch, 2008), for wise and encouraging comments on the topics of section 3 in this paper. Alas, I cannot blame anyone but myself for the remaining infelicities and interpretations of the paper.

## §1. A ‘Fictional’ Preamble

“Here there is a play on words. In English, the word for ‘labyrinth’ is *maze* and for ‘surprise,’ *amazement*. There is a clear semantic connotation as well.

This is the form in which I perceive life: a continual amazement; a continual bifurcation of the labyrinth.”

Borges in Simon (1991), p. 177; italics in the original.

In his dialogue with Simon, initiated and conducted, at the latter’s request, in December, 1970, at Borges’ ‘beautiful high-ceilinged baroque office in the Biblioteca Nacional’, the celebrated Argentinian began by wondering why Simon was interested in having the conversation, in the first place. The above reflection on *the amazement of life’s mazes* was in response to Simon’s answer to this initial question by Borges (*loc.cit.*, p. 176; italics added<sup>1</sup>):

“I want to know how it was that the *labyrinth* entered into your field of vision, into your concepts, so that you incorporated it in your stories.”

In the letter to Borges, requesting a meeting during his visit to Buenos Aires in December, 1970, Simon acknowledged that he had been particularly inspired by “La Biblioteca de Babel” – *The Library of Babel*<sup>2</sup> – to which he had been introduced by Ed Feigenbaum, during one of his regular sojourns at RAND in the academic year 1960/61, through having brought to his attention the collection *Ficciones*.

However, the metaphor of *a journey through a maze* had already played a significant role in Simon’s pioneering formulation of behavioural decision processes by *satisficing boundedly rational agents* (Simon, 1957), thus providing his effective alternative to the orthodox vision of *Olympian, Omniscient*, rational agents (Simon, 1983), optimizing beyond all formal and realistic constraints, even while playing lip service to the framework of ‘constrained optimization’; it was as if engineers were designing perpetual motion machines, despite the elementary laws of even phenomenological thermodynamics.

Many, even enlightened scholars who dispute orthodox formalisations of rationality and the underpinning they provided for human decision making, particularly in the social and humanistic fields, associate ‘*mechanical*’ and ‘*machines*’ with varieties of determinism<sup>3</sup>.

---

<sup>1</sup> Only the part of the dialogue referring to the topic of labyrinths is reproduced in Simon (1991). The full dialogue appears in Simon (1971).

<sup>2</sup> Interestingly one of the more accessible English translations of this fascinating story appears in a book titled **Labyrinths: Selected Stories and Writings** (Borges, 1964).

<sup>3</sup> I am not, here, referring to facile references to deterministic chaos and its possibilities of encapsulating a study of apparently ‘random’ motion.

Simon, following in the noble footsteps of Turing, had begun, just at this time, his later lifelong research focus on trying to model and understand ‘the implication of a belief in the possibility of computer simulation of human thought for free will’ – considered by all shades of authoritative, orthodox, thinking (sic!) to be incompatible with machine modeling of thought – in the specific context of problem solving by bounded rational, satisficing, human decision makers, viewed as *Information Processing Systems*.

This was the way Simon synthesized Turing’s notions of *solubility and unsolvability* in the context of problem solving and Shannon’s parallel, pioneering, work on *information processing and the limits of a transmitting channel’s capacity*. Bringing together, effectively, the way the Halting Problem for Turing *Machines* made it possible to encapsulate the nature of free will in decision making, in the context of (human) problem solving, with the limits of information processing by *mechanical* channels in the sense of Shannon (and later Hamming), was one of Simon’s greatest insights and contributions to what I have come to call *Classical Behavioural Economics* (Kao & Velupillai, 2012).

The mistaken belief that any reference to machines or mechanical precluded any notion, concept or possibility of discussing or formalizing ‘free will’, the indeterminacy of human ‘choice’, whether in the context of finite or infinite – countable, uncountable or whatever – was put to rest in different ways by *Turing, Shannon, Simon and Borges*.

Thus the final answer, this time in answering the question Borges posed Simon, on the nature of his work, Simon was able to state:

“This is the form in which I conceive ***free will***: it resides in the fact that *I* am that which acts when I take a given action. And the fact that something has caused this behavior in no manner make me (the I who acts ***unfree***).

So when we reach a bifurcation in the road of the labyrinth, ‘something’ chooses which branch to take. And the reason for my researches, and the reason why ***labyrinths*** have fascinated me, has been my desire to observe people as they encounter ***bifurcations***<sup>4</sup> and try to understand why they take the road to the right or to the left.”

Simon, 1991, p. 179; bold italics, added.

---

<sup>4</sup> I hope the discerning reader is now strengthened in his appreciation of the caveat I added in the previous footnote!

The rest of this brief paper is structured as follows. In the next section, against the backdrop provided in this ‘fictional’ preamble, a concise description of the way Simon used the metaphor of a journey through a maze – a labyrinth – to formalize his pioneering alternative to orthodox formulations of Olympian, Omniscient, rational behavior; the former in a natural and intrinsic procedural way – one actually travels through the labyrinth, with or without the benefits of possessing *Ariadne’s Thread* – and the latter in a fictitious world of *as if* mongering.

In section 3, inspired by Bloch (2008), I weave a few unpredictable themes, based on Borges’ imagined lives of the librarians in his *Library of Babel*, using Turing’s remarkable theorem on the Halting Problem for Turing Machines that makes nonsense of the varied deprecations of noble claims by Turing, Simon and Borges on thoughts and thinking by simulating machines or minds (whether they are the same, or one can be studied by simulating the other or whatever, remains, in fact, undecidable in any formal sense).

The final, ultra brief section, outlines, speculatively, what I think we as non-orthodox, interdisciplinary economists, can learn, at least in the form of metaphors, from repeated and unceasing reading of Borges’ apparently paradoxical stories – none of which are remotely formal in any mathematical sense, but almost all of which are infused with deep mathematical insights.

## § 2. A Classical Behavioural Economist’s Journey in a Maze

“Classical economic theory assumes that decision makers, whether groups or individuals, know *everything* about the world and use it all to calculate the optimal way to behave. ... *This is a ridiculous view of what goes on.*

To go into a firm and evaluate the actual decision-making process, you must find out what information they have, *choose* to focus on, and how they *actually process that information*. That’s what I’ve been doing all these years. *That’s why my AI work is a natural continuation of what I did earlier in economics*. It’s all an attempt to see *how decision making works*: first at the individual level--how is it possible to solve problems with an instrument like a human brain?--and at the group level, although I’ve never gotten back to that level.”  
Simon, 1994; italics added.

By his own admission (see Part II, Simon, 1991), 1956, was a watershed year that saw the publication of the article in which the maze metaphor was utilized effectively (Simon, 1956), to launch the boundedly rational agent, satisficing, as an Information Processing System solving problems i.e., towards the monumental treatise on *Human Problem Solving* (Newell & Simon, 1972). The Olympian

rational agent of orthodox theory – even the ostensibly extended behavioural agent of *Modern Behavioural Economics* of Ward Edwards who, also ‘emerged’ at about the same time (Kao & Velupillai, *op.cit.*) – who, in any case, *almost* never played any role in Simon’s work before the watershed year, completely disappeared from the world of Simon’s behavioural sciences from that time onwards. There was never any infinite horizon, utility maximizing paradigm in Simon’s contribution after the maze metaphor helped him launch his (classical) boundedly rational, satisficing, behavioural agent, procedurally searching for an acceptable path along the labyrinth that was the life of decision making.

As he characterized it:

“In my 1956 paper, ‘Rational Choice and the Structure of the Environment,’ I wove around *the metaphor of the maze* a formal model of how an organism (a person?) could meet a multiplicity of needs and wants at a satisfactory level and survive without drawing upon superhuman powers of intelligence and computation. The model provided a practicable design for a creature of *bounded rationality*, as all we creatures are.”

Simon, 1991, p. 175; italics added.

From the weaving of a formal model of decision making, around the metaphor of a maze, Simon next took the imaginative – even audacious – step of writing a story: *The Apple: A Story of a Maze*. It illustrated his theory of human decision making, as a process of learning heuristics, eventually formalisable as algorithms, during a the life of a reasonably ‘intelligent’ – albeit isolated from other human interactions – agent, traversing *endlessly* through the mazes of a labyrinth.

The structure of the maze that housed Hugo, the protagonist in *Apple*, had great mathematical similarities to the *Library of Babel*, especially its never-ending potential paths and the unmentioned length of life that Hugo could expect to live, if – indeed – life and death made any sense to him. In other words, it is not clear whether Hugo had any perception of ageing, although he obviously felt the passage of diurnal time.

Hugo’s focus, in addition to the internal ruminations that result in learning from his daily experiences of journeying through the maze that makes up the labyrinth – although there is Minotaur he is searching for, to slay, ultimately - is the environment. Here, Hugo is assumed to have a single, fixed, aspiration level for food, but as he realizes that there are different ways to satisfy this aspiration, he develops a certain sophistication in his taste (not realizing, of course, that he may or may not have intrinsic, mutable, taste buds with which he may or may not have been genetically endowed). But, the alternative ‘menus’ are located in such a way that Hugo has to walk in the maze, where there are branches at each bifurcation. Hugo has a finite number of possible alternative routes to choose, from any bifurcation, which leads him to one or another of a finite variety of locations of alternative menus.

Over time, Hugo develops likes and dislikes – what the economist may refer to as ‘preferences’, so they are not given at the outset, as in the case of the neoclassical, Olympian, agent.

The ‘evolving, taste is combined with the constraint that Hugo’s vision – both for intrinsic organic reasons and extrinsic structural reasons - is limited and therefore his vision of the alternatives from any bifurcation is more or less constrained. However, if his vision permits the sight of menus, then Hugo knows the way to reach the food. Hugo has to satisfy hunger and thirst for purely survival purposes which, presumably, is signaled by internal biological mechanisms and there is a maximal number of moves he can make after eating before his energy runs out. It is not clear whether or how exactly he learns these constraints, but learn he does and these ‘loose ends’, in the novelette, allow for the kind of imagination that the Borges story caters for, copiously.

There is no wonder, then, that Simon, when he read *The Library of Babel* (around 1960/61), felt a strong affinity with the richness of the possibilities of indefinite paths, indeterminate ageing, uncountable infinities and other surreal imagined realities that enriched the Borges story. The obvious difference between the two, of course, was that Borges left these imagined realities in the realm of the magic of the mind; Simon spent the rest of his life trying to tame them, by modeling the many indeterminacies, indefiniteness and infinities with imaginative mathematics, for the most part those that Turing devised, with other – yet equally mind-boggling – inspirations at its bases.

To a few of them, I now turn.

### § 3. Turing’s *Librarian in Borges’ Library of Babel*<sup>5</sup>

“Like all men of the Library, I have traveled in my youth; I have wondered in search of a book, perhaps the catalogue of catalogues: now that my eyes can hardly decipher what I write, I am preparing to die just a few leagues from the hexagons where I was born.”  
Borges, 1964, p.52

In his *paean* to Bloch’s sustained, almost lyrical, mathematical rendering of *The Library of Babel*, the extraordinary magic realistic essay by Borges, formally written as a book review, Dan King (2010, p. 418), perceptively observed:

“In more uncharted terrain, Bloch closes his mathematical analysis of [*Library of Babel*] by weaving connections to the works of Turing and Gödel. He argues that the combined lives of the librarians endlessly searching the volumes of the Library can, in a sense, be regarded as homomorphic to the operations of a Turing Machine.”

---

<sup>5</sup> I am using James Irby’s translation in Borges (1964).

Bloch's characterization of the activities of a single librarian<sup>6</sup> - there are, obviously<sup>7</sup>, a *countable infinity* of them, since there are *at least* a countable infinity of hexagons and, according to the *Library of Babel* (footnote 1, p. 54), 'there was a [librarian] for every three hexagons'<sup>8</sup> - as those of the computation by a Turing Machine are persuasively described (*ibid*, p.124, last paragraph). The characterization is schematized as a formal (symbolic) computational procedure by a Turing Machine, with a defined halting state - i.e., death ('by suicide and pulmonary diseases?').

However, the Turing Machine interpretation of the life and times of a Borgesian Librarian, in the *Library of Babel*, remains *incomplete* (sic!) without the addition of the following 'Lemma':

Lemma: It is (algorithmically) undecidable when (and *whether*) any particular librarian will expire.

Proof: Due to the *undecidability*<sup>9</sup> of the *halting problem for Turing Machines*.

I should add one 'surreal' (*pace* Knuth, 1974) mathematical note to substantiate some of the implicit mathematical assumptions in the notes above. The existence of a *countable infinity* of hexagons, in finite space, is easily formalized within one or another form of nonstandard analysis<sup>10</sup>, as clearly indicated by Bloch (see, in particular, pp. 53-4, *ibid*).

Two other, distinctly *non-surreal*, notes refer to possible *non-consistencies*<sup>11</sup> in Borges. One, when he refers to the books in the *Library of Babel* having been composed using only *twenty-*

---

<sup>6</sup> I do not think, contrary to King's interpretation of Bloch, that this characterization is about 'the *combined lives* of the librarians', but that of *one* arbitrary librarian. I am happy that Professor Bloch confirms my interpretation in an e-mail to me of 13 December, 2012.

<sup>7</sup> I am not sure 'obviously' is completely warranted here; I suspect there is some ambiguity in the original Borges text which makes it possible to think otherwise.

<sup>8</sup> Although Borges adds the caveat (*loc.cit*): 'Suicide and pulmonary diseases have destroyed that proportion', this does *not* mean the existence of *any less than* a countable infinity of librarians, since only the 'proportions' are 'destroyed', not annihilated.

<sup>9</sup> I add this 'lemma', invoking also the use of the words 'incomplete' and 'undecidable' to substantiate Bloch's otherwise seemingly unnecessary reference to Gödel (*ibid*, p. 121).

<sup>10</sup> My own preference in this regard, particularly for reasons of easy mathematical compatibility with the computability requirements above, would be via '*Smooth*' *Infinitesimal Analysis*, with its underpinning in both intuitionistic logic and category theory (cf., for eg., Bell, 1998).

<sup>11</sup> I choose this word, avoiding the more familiar 'inconsistency' because, in the spirit of Bloch's lyricism and King's paean, I do not want to imagine Borges being subject to ordinary strictures, mathematical or otherwise.

*five orthographical symbols* (*ibid*, p, 53), ruling out the use of ‘digits or capital letters in the original manuscripts’. Yet, on the same page he observes:

“One [book] which my father saw in a hexagon on circuit fifteen ninety-four was made up of the letters MCV, perversely repeated from the first line to the last.”

The second possible non-consistency is when he claims that the number of books, since they are ‘made up of the same elements: the space, the period, the comma, the twenty-two letters of the alphabet’, are ‘not infinite’ (*ibid*, p. 54). However, this has to be an *impossible* conclusion, given the premises! An elementary enumeration, even constructively enabled, would convince a reader, in particular, of Borges and Bloch, that the *listable* combinations of the ‘twenty-odd orthographical symbols’ is, in fact, countably infinite.

However, an important observation by Bloch<sup>12</sup> should be taken into consideration here:

“My guess would be that [Borges] meant something like, ‘The number of volumes with *distinct orderings from the finite alphabet is itself finite*, due to the fact that each volume has a prescribed number of symbols in it.’ This would entail a finite number of distinct volumes; the  $25^{1,312,000}$  calculated by many besides myself. *If one allowed multi-volume sets of books with unlimited repetition*, then you are, of course, correct.

These non-consistencies and ambiguities are absolute trivialities in Bloch’s lyrical rendering of **The Unimaginable Mathematics of Borges’ Library of Babel**.

#### § 4. Mathematising Imagined Worlds

“It is not uncommon for a scientist to be interested in Borges . . . . . Deep and varied connections sprang out as we read about *aleph* – the *cardinality* of infinity<sup>13</sup> . . . . – about *forking paths* that lead to parallel universes . . . . Or about *an infinite library* that in the end turns out to have the same contents as a single ‘book of sand,’ whose number of pages is a *continuum*.”

Quiroga, 2012, pp. 4-5; italics added

Quiroga’s fascinating book lists an impressive number of scientists – including mathematicians<sup>14</sup> – who have read and interpreted the imagined unimaginable entities and structures in Borges’ many stories<sup>15</sup>.

---

<sup>12</sup> In the personal note to me referred to above, in footnote 5.

<sup>13</sup> I am not sure this is a rigorously correct assertion.

<sup>14</sup> Odifreddi’s wonderful thoughts (Odifreddi, 1997) emphasises the role of self-referencing in many of the Borges stories, interpreted metamathematically. In particular, in **Tlön, Uqbar and Orbis Tertius**. I have myself used **Kafka and his Precursors** for years in my economic writings.

Do we – are we able to – encapsulate the Borgesian world of imagined unimaginable in any kind of formal mathematics? It is clear from the remarks and substantiated observations in Bloch, Odifreddi, Quiroga and Simon, that Borges had more than a nodding acquaintance with at least some aspects of the foundations of mathematics and mathematical philosophy. But he moulded the deep mathematical concepts he worked with, predominantly for purposes of constructing and representing literary paradoxes, from which to extract lessons for the mind – and to represent deep and difficult ambiguities.

As far as I am concerned, the role of *paradoxes* and *ambiguities* in pushing the frontiers of economic formalisations have been less than felicitous. They have been confined to what I consider trivial examples, predominantly characterized by work in Modern Behavioural Economics, which I have also referred to as a field indulging in ‘anomaly mongering’, resolved *by ad hoc* appeals to a plethora of mathematical concepts. They play, at best, the role of counter-examples in mathematics, but without the discipline provided by an *Ansatz* or a clear understanding of the function of *thought-experiments* in dissecting and ‘repairing’ paradoxes or clarifying ambiguities.

I believe Simon’s understanding of the vision that was behind the narratives Borges constructed is the best available, at least for now (Simon, 1991, p. 179; italics added):

“So Borges denied that there was an abstract model underlying ‘The Library of Babel’ or ‘The Garden of Paths that Fork’. *He wrote stories; he did not instantiate models. He was a teller of tales.*”

The message, then, at least for economics, seems to be that we, too, should try to ‘write stories’, be ‘tellers of tales’, without this mad rush into ‘instantiating models.’

---

<sup>15</sup> Just today, New Year’s day, 2013, Ming Hai, my former student at the People’s University of China (a quarter of a century ago) wrote me wondering whether I should not also consider the relevance of the story of *The Circular Ruins* for my philosophy of economics! Quiroga’s book is a sustained study of the relevance of **Funes the Memorios** for his own field of research, broadly conceived as the neurosciences.

## References

- Bell, John. L (1998), **A Primer of Infinitesimal Analysis**, Cambridge University press, Cambridge.
- Bloch, William Goldbloom (2008), **The Unimaginable Mathematics of Borges' Library of Babel**, Oxford University Press, Oxford.
- Borges, Jorge Luis (1964), **Labyrinths: Selected Stories & Writings**, New Directions, New York.
- Kao, Ying-Fang & K. Vela Velupillai ((2012), *Origins and Pioneers of Behavioural Economics*, **Interdisciplinary Journal of Economics and Business Law**, 1(3), 47-73.
- King, Dan (2010), *Book Review of Bloch (2008), The College Mathematics Journal*, Vol. 41, No. 5, November, pp. 416-418.
- Knuth, Donald. E (1974), **Surreal Numbers: A Mathematical Novelette**, Addison-Wesley Publishing Company, Inc., London.
- Newell, Allen & Herbert. A Simon (1972), **Human Problem Solving**, Prentice-Hall, INC, Englewood Cliffs, NJ.
- Odifreddi, Piergiorgio (1997), *Un Matematico Legge Borges*, Text of a Lecture delivered at a conference on **Science and Art**, held in Trieste, November, 1996.
- Quiroga, Rodrigo Quian (2012), **Borges and Memory: Encounters with the Human Brain**, The MIT Press, Cambridge, Massachusetts.
- Simon, Herbert. A (1956), *Rational Choice and the Structure of the Environment*, **Psychological Review**, 63(2), 129-38.
- Simon, Herbert. A (1957), **Models of Man**, John Wiley & Sons, Inc.
- Simon, Herbert. A (1971), *Primera Plana va más lejos con Herbert Simon y Jorge Luis Borges*, **Primera Plana**, 414, 5 Gennaio, pp. 42-45.
- Simon, Herbert. A (1983), **Reason in Human Affairs**, Basil Blackwell, Oxford.
- Simon, Herbert. A (1991), **Models of My Life**, The MIT Press, Cambridge, Massachusetts.
- Simon, Herbert. A (1994), *Herbert. A Simon: Thinking Machines: Interviewed June 1994 by Doug Stewart*, **OMNI Interview**, June.